LEt-SNE: A Hybrid Approach to Data Embedding and Visualization Of Hyperspectral Imagery

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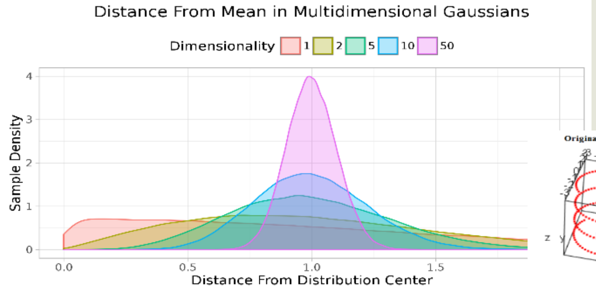
Agenda

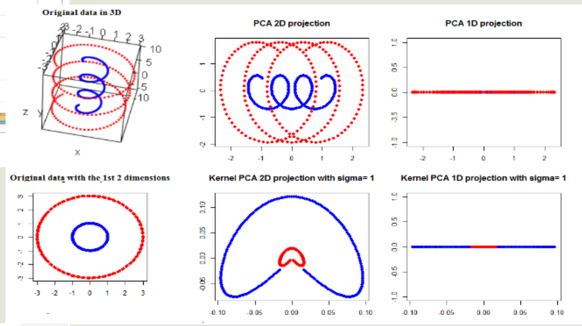
Problem Statement
- Dimensionality Reduction
- Significance: Curse Of Dimensionality

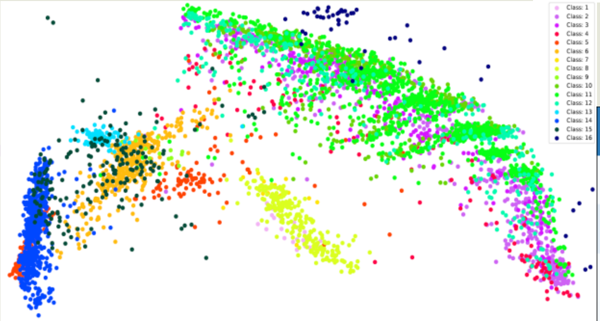
Literature Review
- Classical Methods
- Gradient Descent Methods

Methodology
- Visualization
- Clustering

Results
- Accuracy: Kappa Score
- Confusion Matrix







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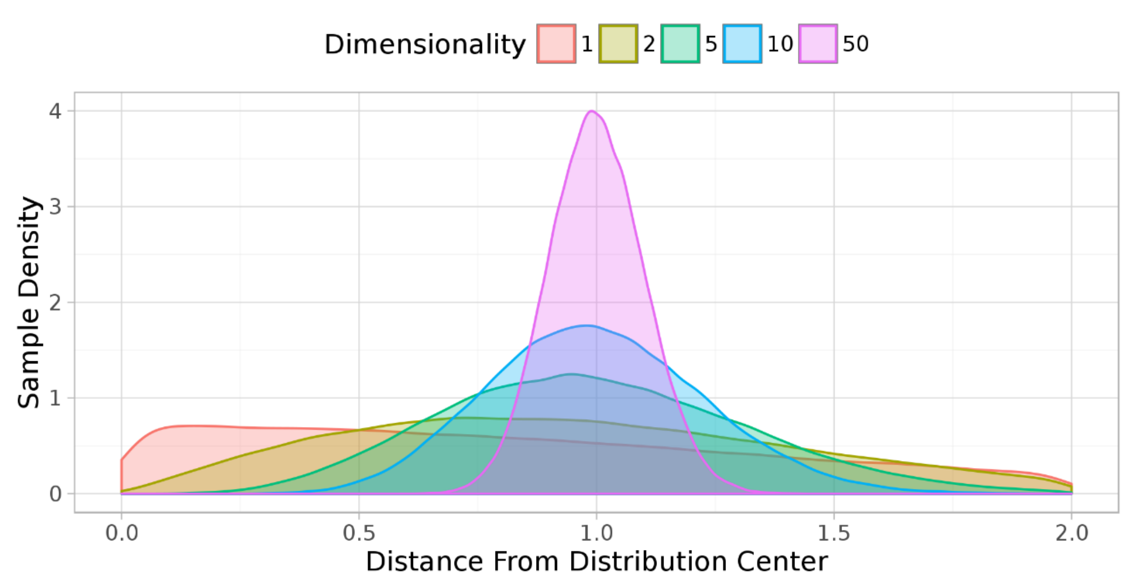
Dimensionality Reduction – Motivation ?



Hamner, B., 2016, Kaggle Blog

$$\lim_{d\to\infty} \mathbb{E}\left(\frac{dist_{max}(d) - dist_{min}(d)}{dist_{min}(d)}\right) \to 0$$

Curse Of Dimensionality

VC Dimension - Overfitting

VC_{dim}(NeuralNet) = $O(WL \log W)$ | $W = \#weights$ $L = \#layers$

Hyperspectral Imagery

Dimensionality : 100-200 | Correlated Bands

[Bartlett et al., "Nearly-tight VC-dimension and Pseudodimension Bounds for Piecewise Linear Neural Networks", Journal of Machine Learning Research]

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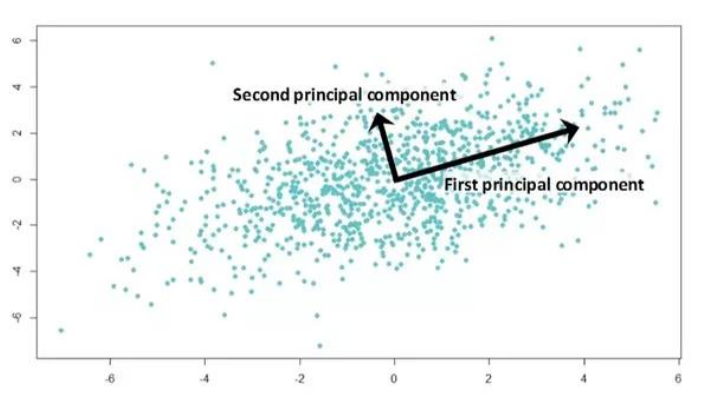
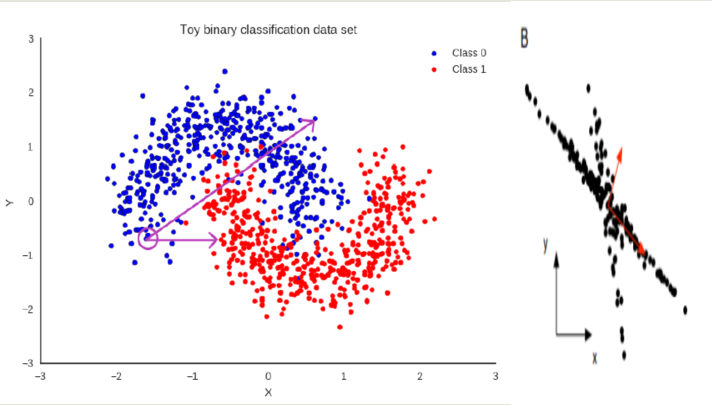
Literature Review

Principal Component Analysis and Linear Discriminant Analysis

PCA

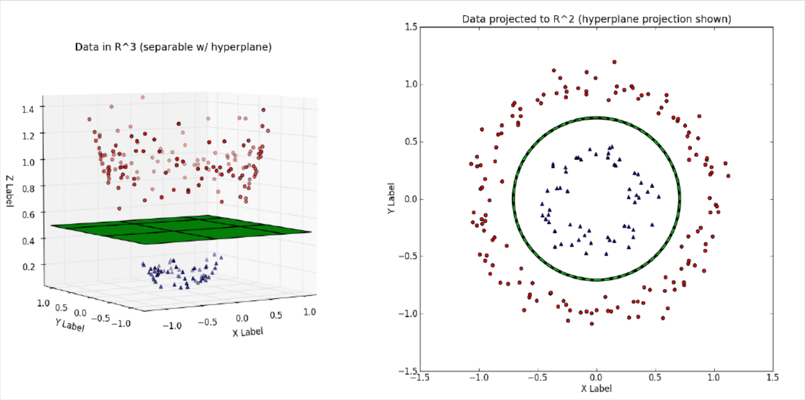


https://medium.com/@sadatnazrul/the-dos-and-donts-of-principal-component-analysis-7c2e9dc8cc48

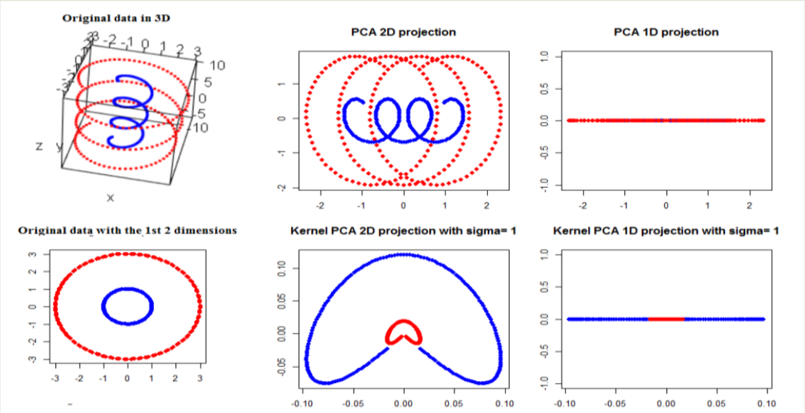


Shlens, J., 2014, “A tutorial on principal component analysis,”

Kernel-PCA

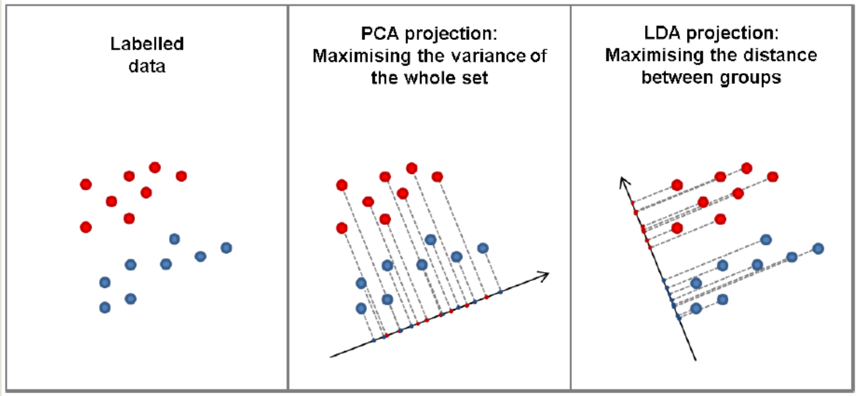


https://www.eric-kim.net/eric-kim-net/posts/1/kernel_trick.html

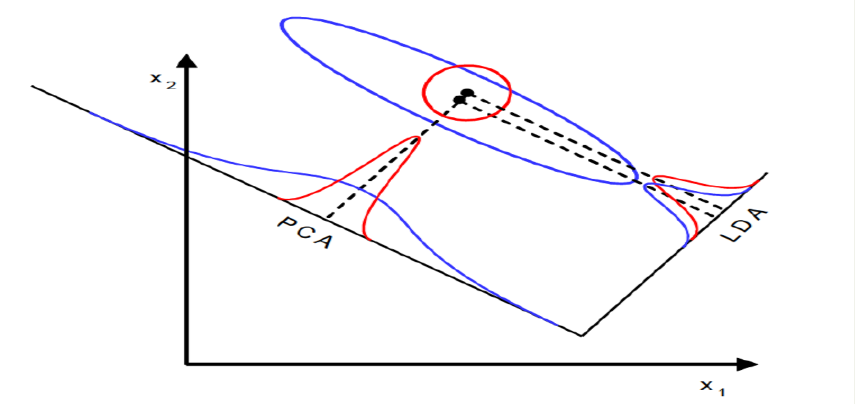


https://rpubs.com/sandipan/197468

LDA



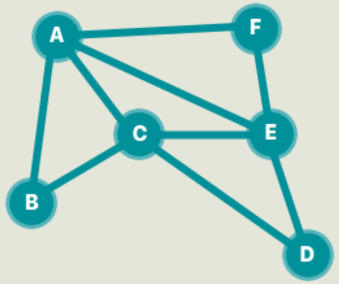
https://www.idtools.com.au/classification-nir-spectra-linear-discriminant-analysis-python/



Literature Review

Laplacian Eigenmaps and Locally Linear Embedding

Laplacian Eigenmaps



=

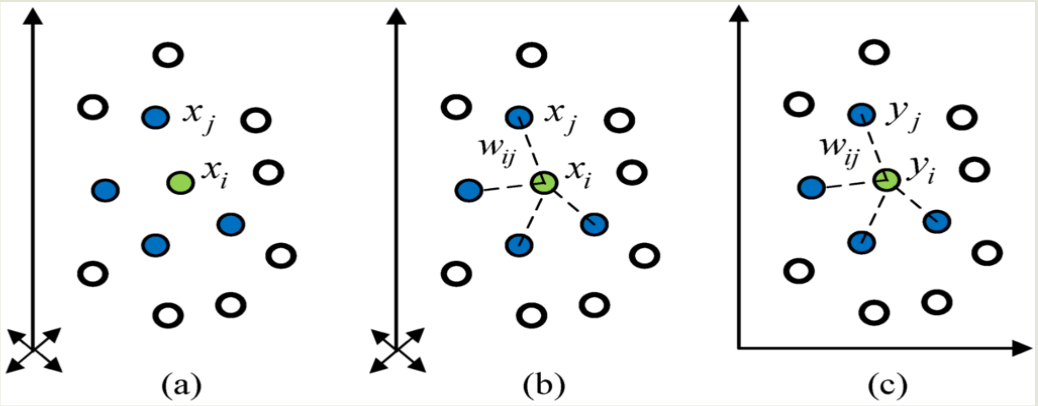
	A	B	C	D	E	F
A	0	1	1	0	1	1
B	1	0	1	0	0	0
C	1	1	0	1	1	0
D	0	0	1	0	1	0
E	1	0	1	1	0	0
F	1	0	0	0	1	0

V: {A,B,C,D,E,F}

E: {AB,AC,AF,BC,CD,CE,DE,EF}

https://towardsdatascience.com/graph-theory-set-matrix-notation-7dfb04b8ed24

Locally Linear Embedding



Xiang Wang et al., "Bearing Fault Diagnosis Based on Statistical Locally Linear Embedding", Sensors

$$\mathcal{J}(y) = \sum_{i,j} (y_i - y_j)^2 A_{ij}$$

Adjacency Matrix

Embeddings

$$\underset{\substack{y^T D y = I \\ y^T D 1 = 0}}{argmin}\ y^T L y$$

Local Structure > Global Structure

$$\mathcal{E}(W) = \sum_i |\, x_i - \sum_j W_{ij} x_j \,|^2$$

Weight Matrix

Original samples

$$\underset{y}{argmin} \sum_i |\, y_i - \sum_j W_{ij} y_j \,|^2$$

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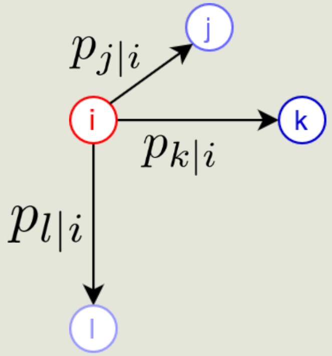
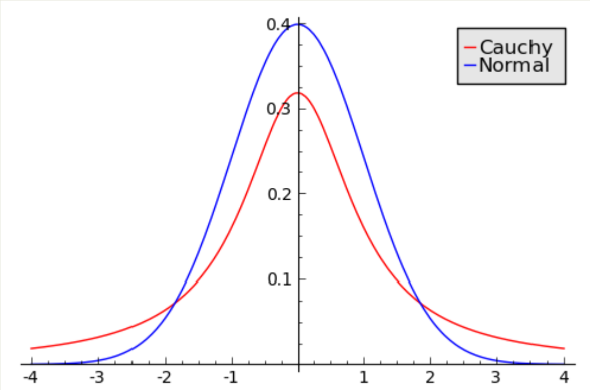
Literature Review
t-SNE and UMAP

t-SNE

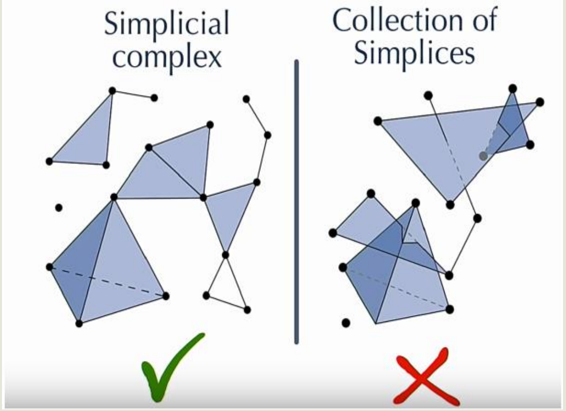


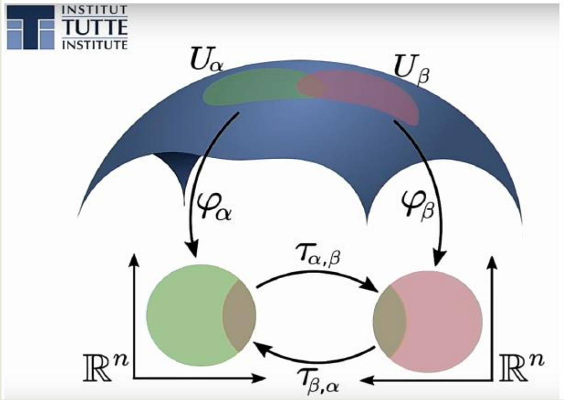
$$p_{j|i} = \frac{exp(-\| x_i - x_j \|^2 / 2\sigma_i^2)}{\sum_{k \neq i} exp(-\| x_i - x_k \|^2 / 2\sigma_i^2)}$$

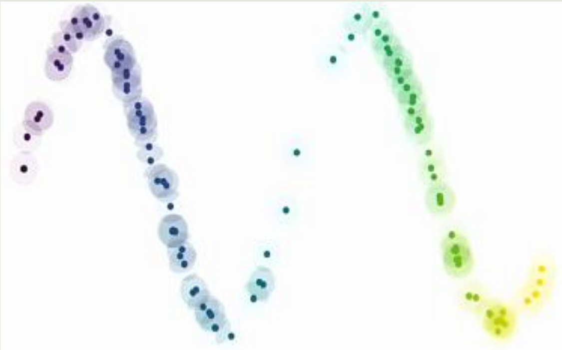


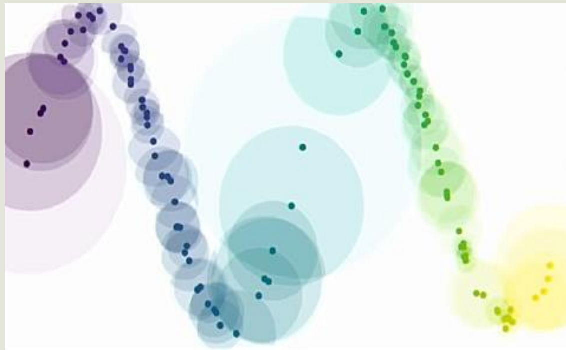
https://stats.stackexchange.com/a/285020

UMAP









https://www.youtube.com/watch?v=nq6iPZVUxZU

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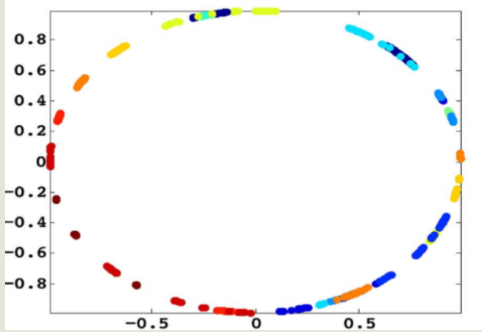
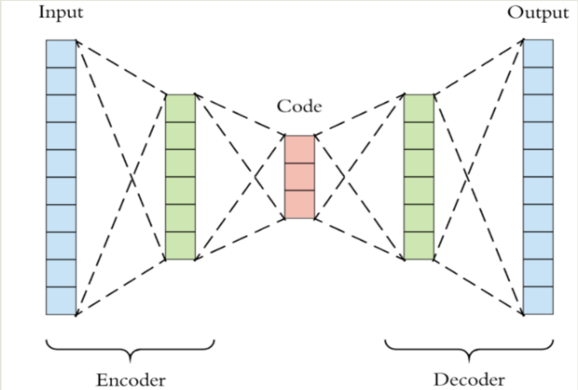
s-SNE



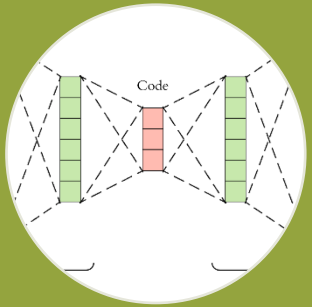
Lunga and Ersoy, "Spherical Stochastic Neighbor Embedding of Hyperspectral Data" IEEE TGRS, Feb. 2013

Autoencoders



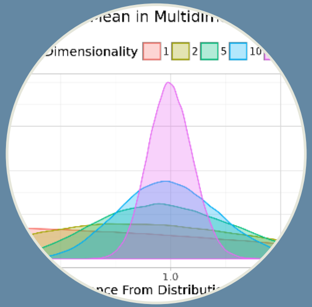
https://medium.com/red-buffer/autoencoders-guide-and-code-in-tensorflow-2-0-a4101571ce56

Why Propose A New Method?



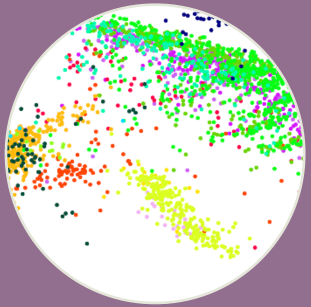
Parameterization

Multilayer Perceptron



Curse Of Dimensionality

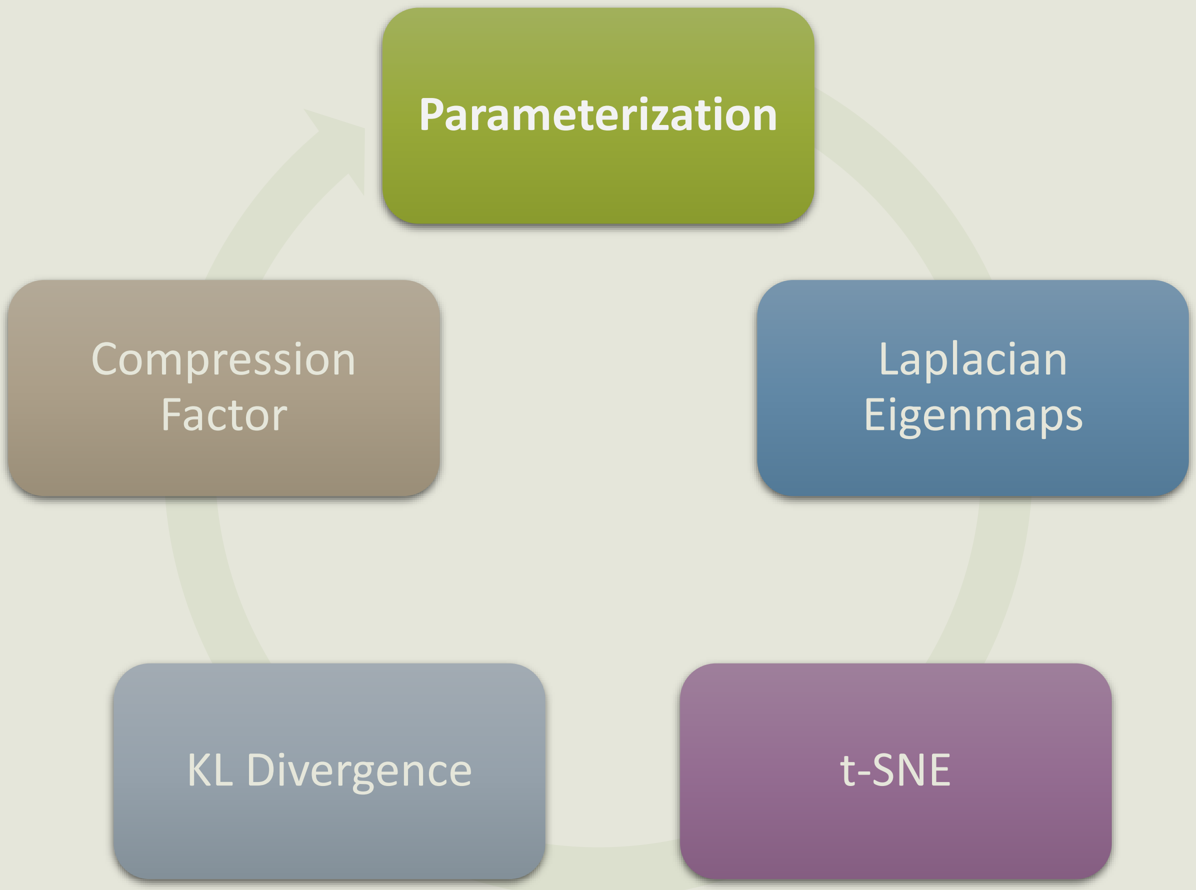
Compression Factor

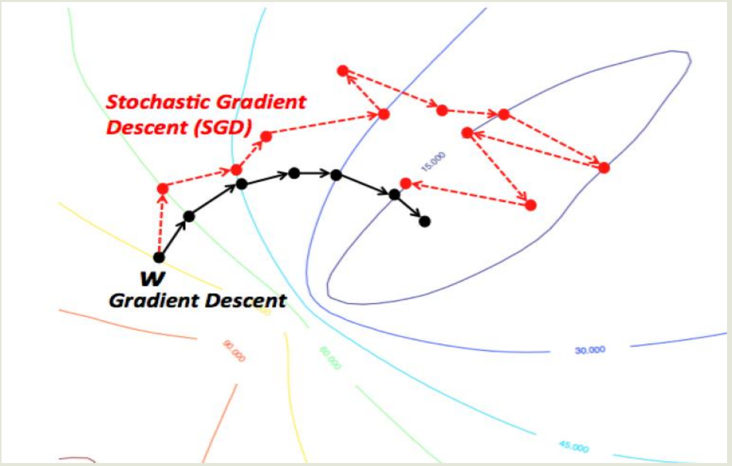


Visualization and Clustering

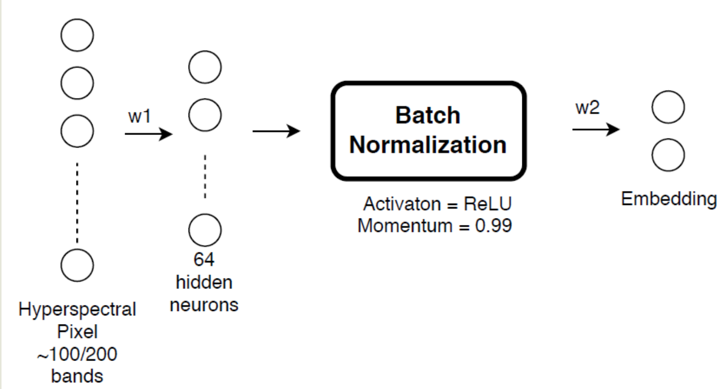
KL Divergence

LEt-SNE
Designing the algorithm

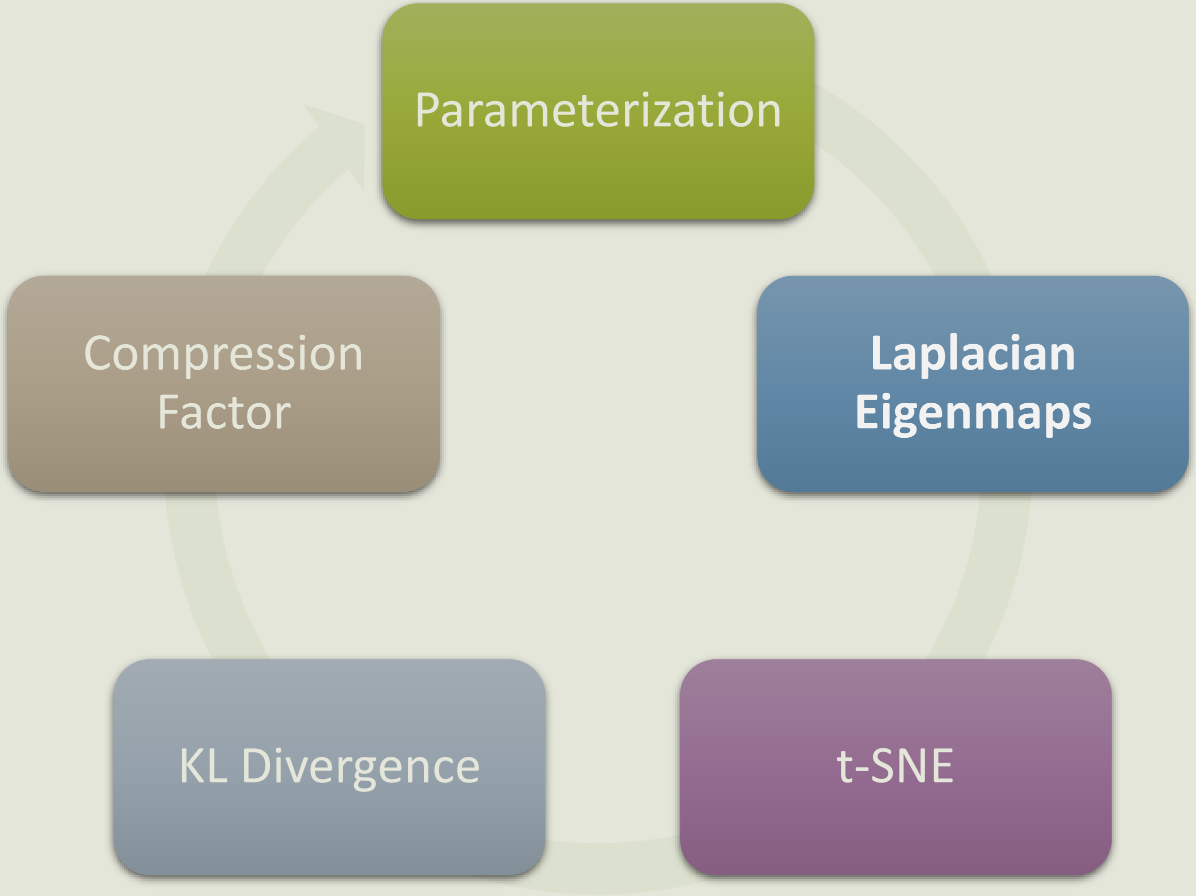




https://wikidocs.net/3413



LEt-SNE
Designing the algorithm



$$Y = f(X, w)$$

Multilayer perceptron X parameterized by w to yield encodings Y

Minimization Objective … ?

$$\nabla_w Y^T L Y$$

✖ $||w|| \downarrow$

Minimization objective … ?

$$\nabla_w Y^T L Y \quad Y^T D Y = I$$

✖ Failed to converge

Ensure grouping of embeddings!

LEt-SNE

Designing the algorithm

Parameterization

Laplacian Eigenmaps

t-SNE

KL Divergence

Compression Factor

$$p_{j|i} = \frac{exp(-\| x_i - x_j \|^2 / 2\sigma_i^2)}{\sum_{k \neq i} exp(-\| x_i - x_k \|^2 / 2\sigma_i^2)}$$

$$q_{j|i} = \frac{exp(-\| y_i - y_j \|^2)}{\sum_{k \neq i} exp(-\| y_i - y_k \|^2)}$$

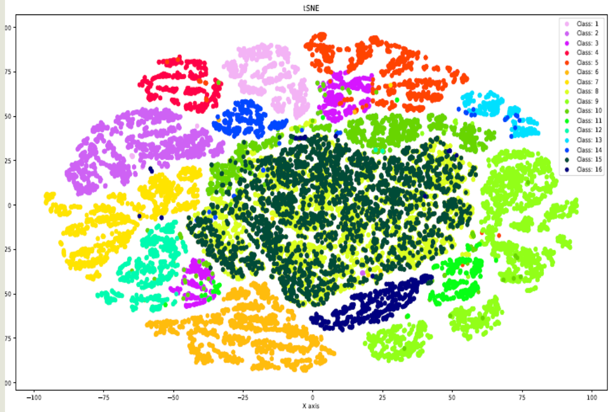
Curse of Dimensionality …

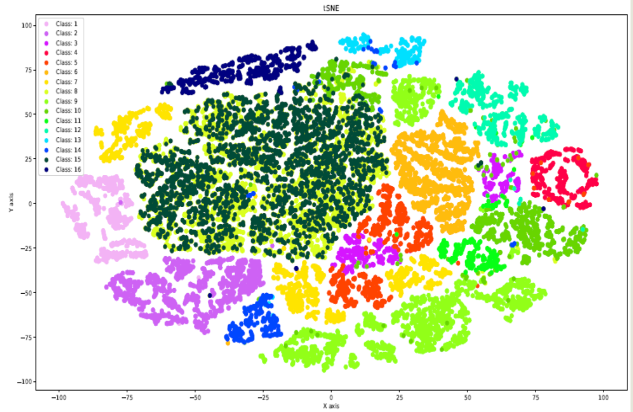
$$p_{j|i} \approx 1/|X|$$

Countering Laplacian Eigenmaps

Keeping embeddings apart

$$q_{j|i} \approx 1/|Y|$$





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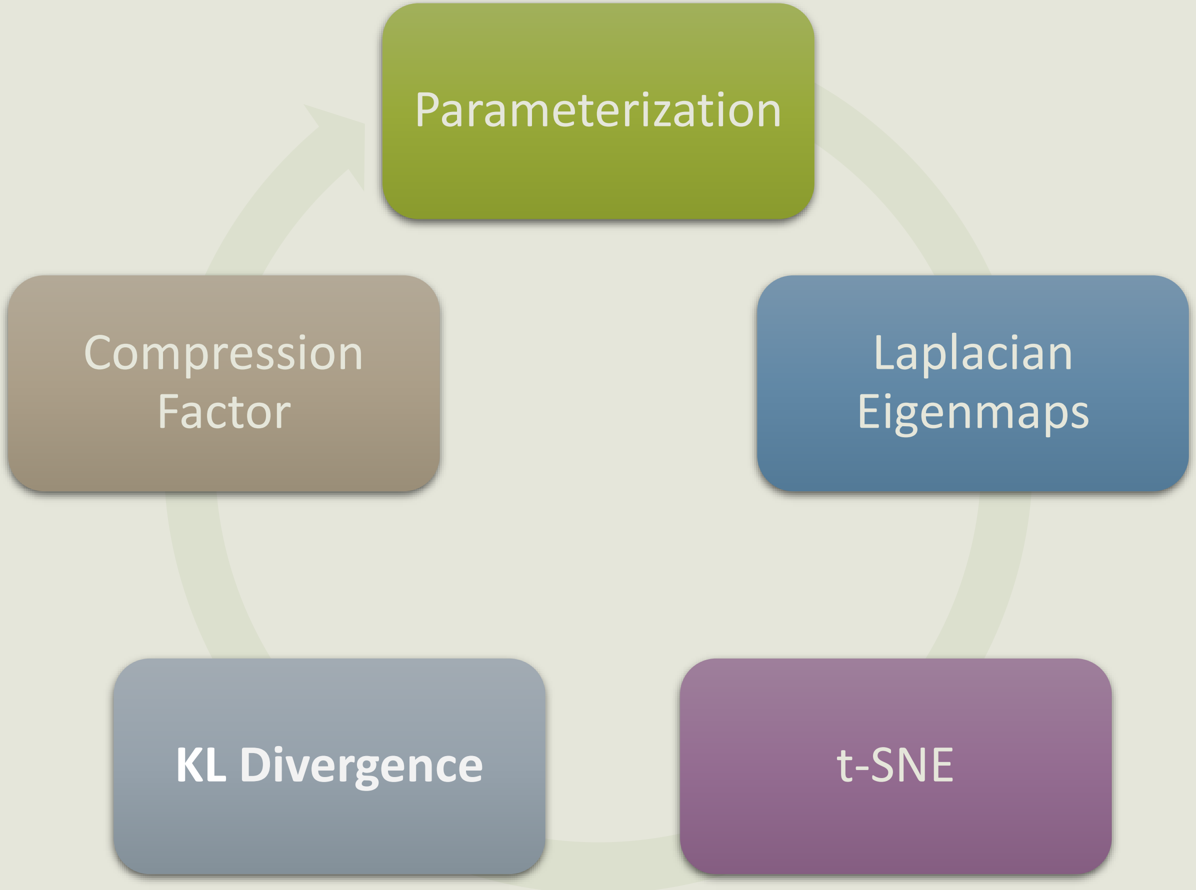
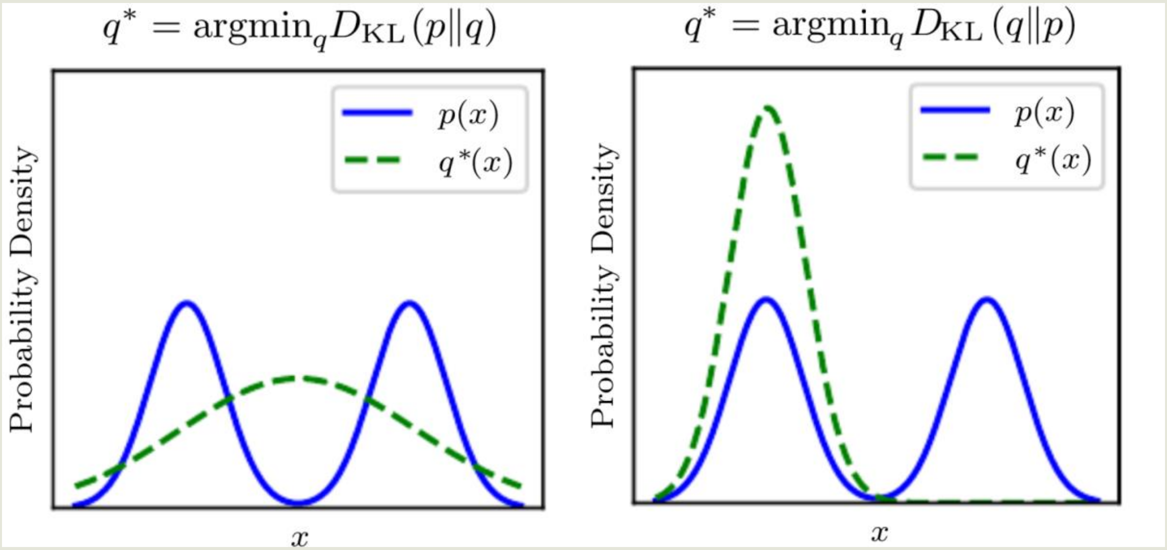
LEt-SNE
Designing the algorithm





Goodfellow et al., "Deep Learning"

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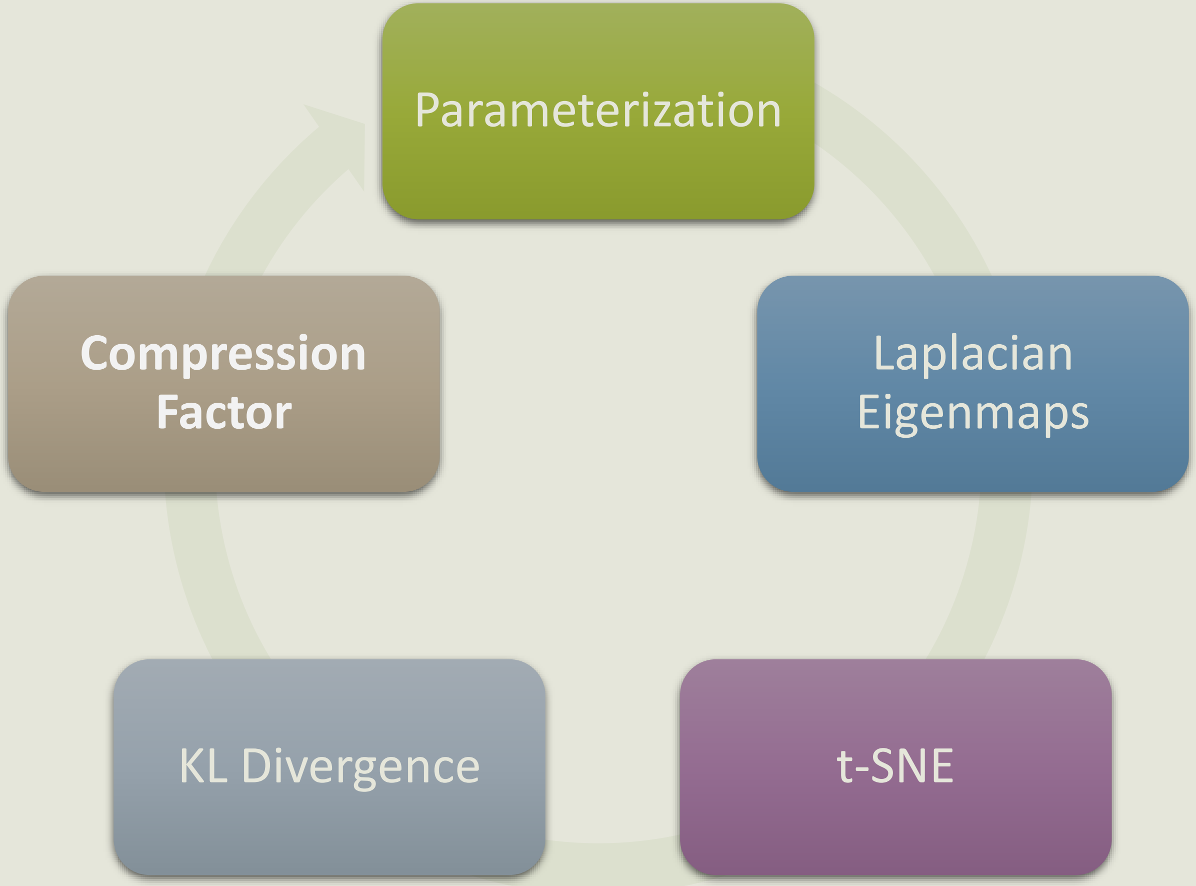
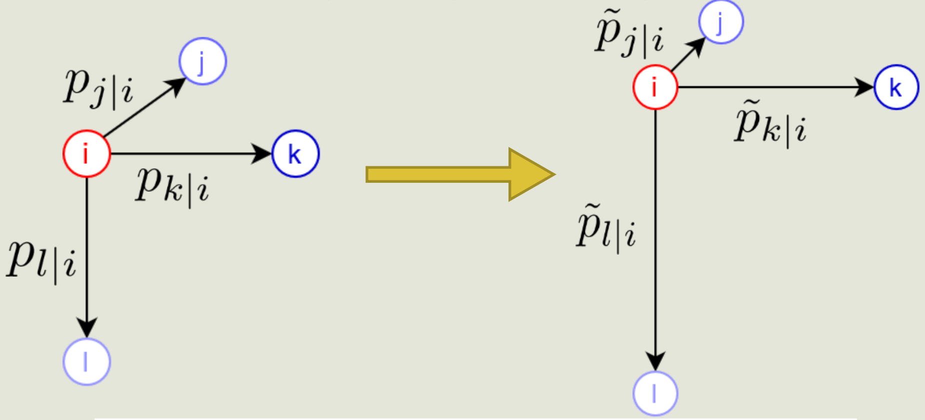
LEt-SNE
Designing the algorithm

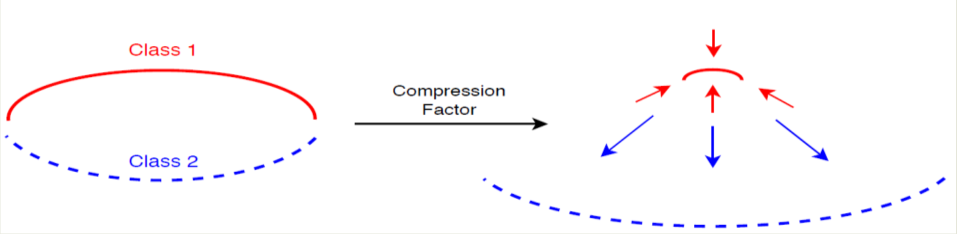


Illusion of manipulating inter-sample distances!



$$\tilde{p}_{j|i} = \frac{p_{j|i} * \{(CF-1) * \mathcal{A}_{ij} + 1\}}{\sum_j p_{j|i} * \{(CF-1) * \mathcal{A}_{ij} + 1\}}$$

Adjacency Matrix
Compression Factor



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LEt-SNE

Stitching the Components

Manifold Visualization

Adjacency Matrix using top-k neighbours

$$arg \min_{w} \mathbb{E}_x \left(\mathcal{Y}^T \mathcal{L} \mathcal{Y} + \lambda \sum_{i,j} \tilde{p}_{i|j} log \frac{\tilde{p}_{i|j}}{q_{i|j}} \right)$$

Laplacian Eigenmaps

t-SNE

Compression Factor < 10

Clustering With Labels

Adjacency Matrix based on Class Labels

$$arg \min_{w} \mathbb{E}_x \left(Y^T \mathcal{L} Y + \lambda \sum_{(i,j)} q_{i|j} log \frac{q_{i|j}}{\tilde{p}_{i|j}} \right)$$

$KL(p||q)$ $\quad$ $KL(q||p)$

Compression Factor > 10

Clustering Without Labels

Adjacency Matrix based on Region Segmentation

$$arg \min_{w} \mathbb{E}_x \left(Y^T \mathcal{L} Y + \lambda \sum_{(i,j)} q_{i|j} log \frac{q_{i|j}}{\tilde{p}_{i|j}} \right)$$

Specific to Hyperspectral Imagery

Compression Factor > 10

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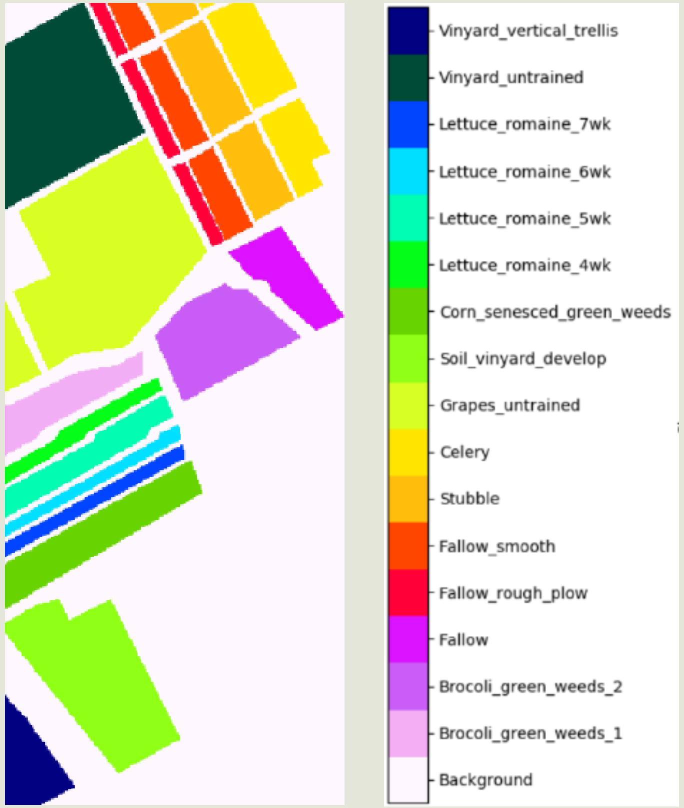
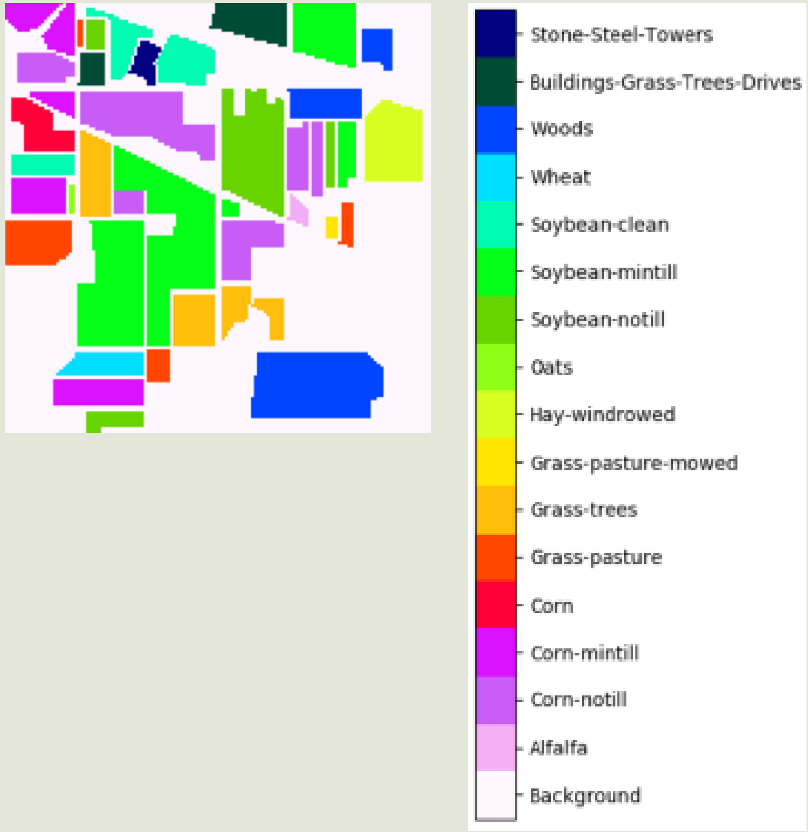
Experimentation
Datasets

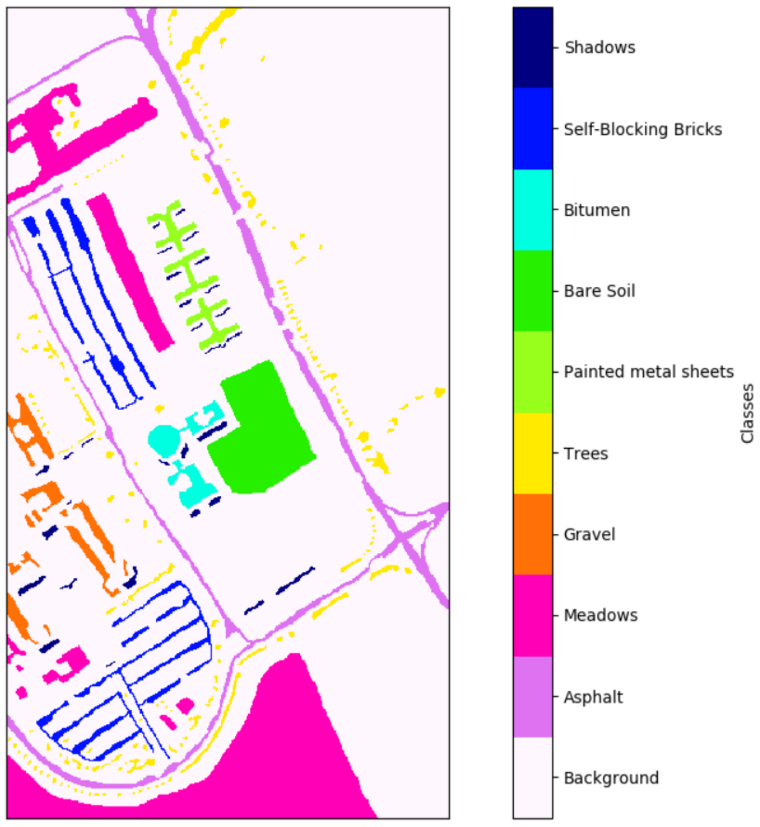
Salinas



Indian Pines



Pavia University



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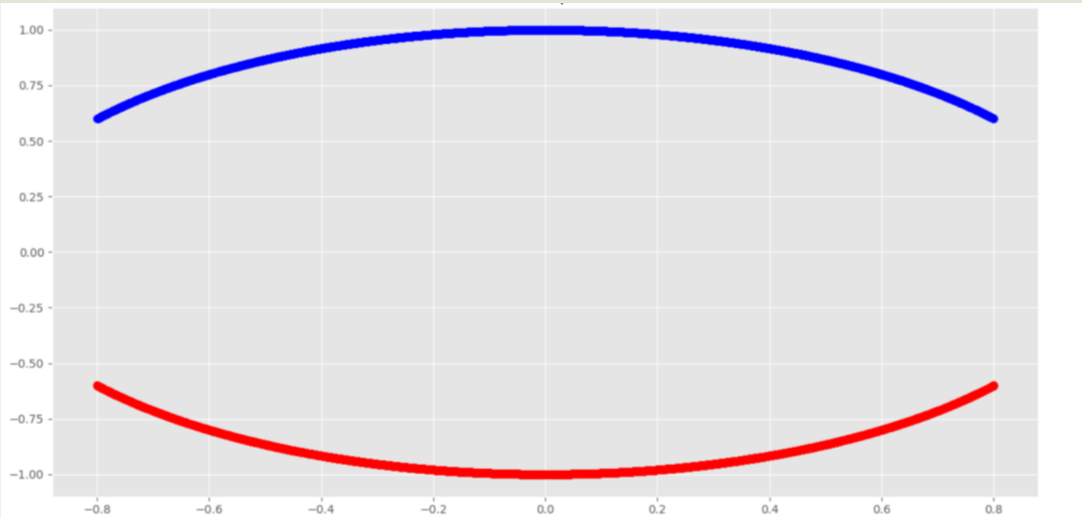
Experimentation
Compression Factor



10,000 samples drawn from $x^2 + y^2 = 1$

Compressing with Labels

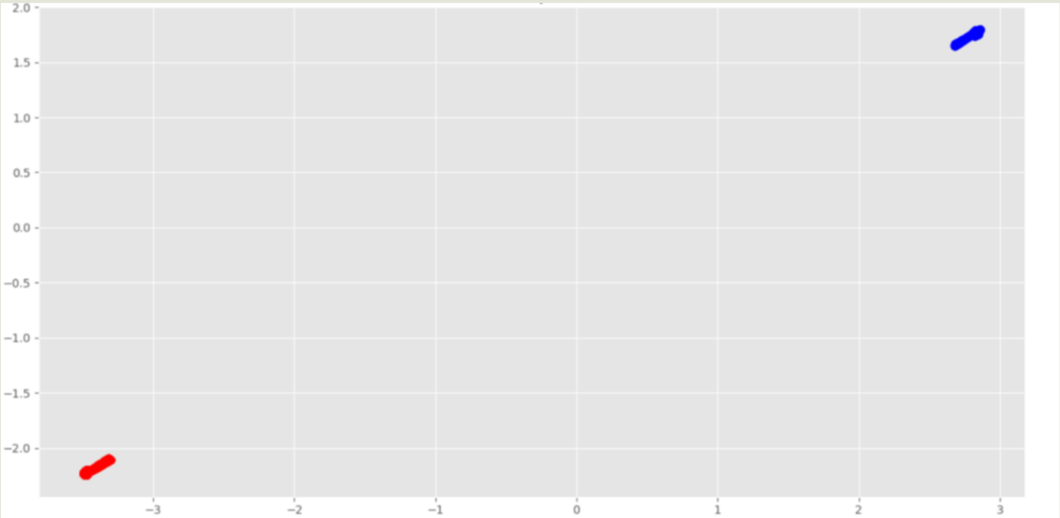
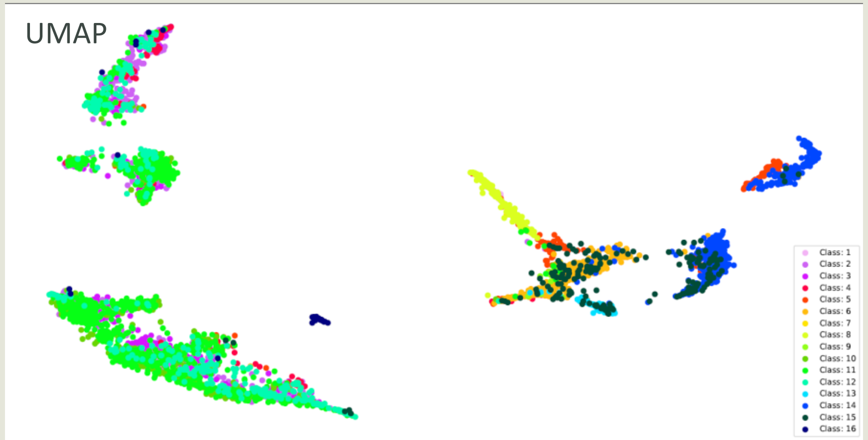
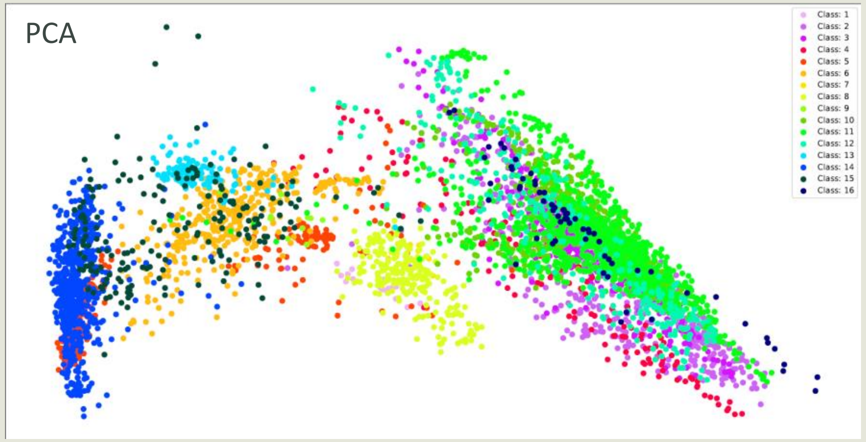


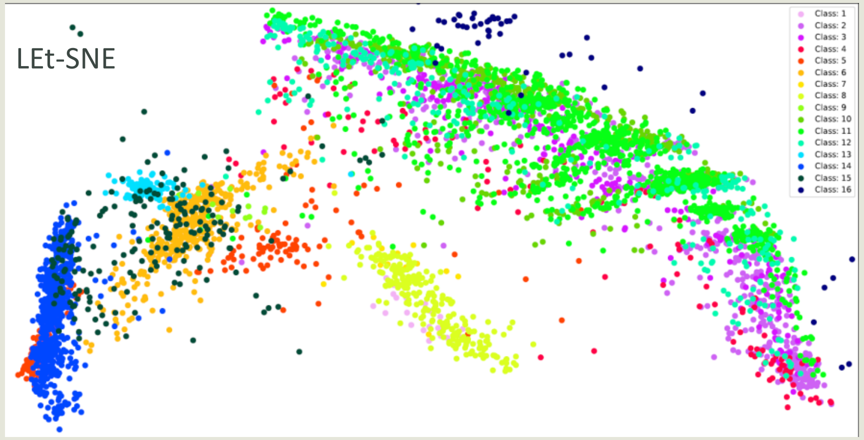
Table 1: Compression Factor: LEt-SNE (sup) with Dimensions = 2

Compression	Indian Pines	Salinas	Pavia
NA	0.4936	0.7877	0.7534
200	**0.6207**	**0.9236**	**0.8594**

Experimentation
Visualization







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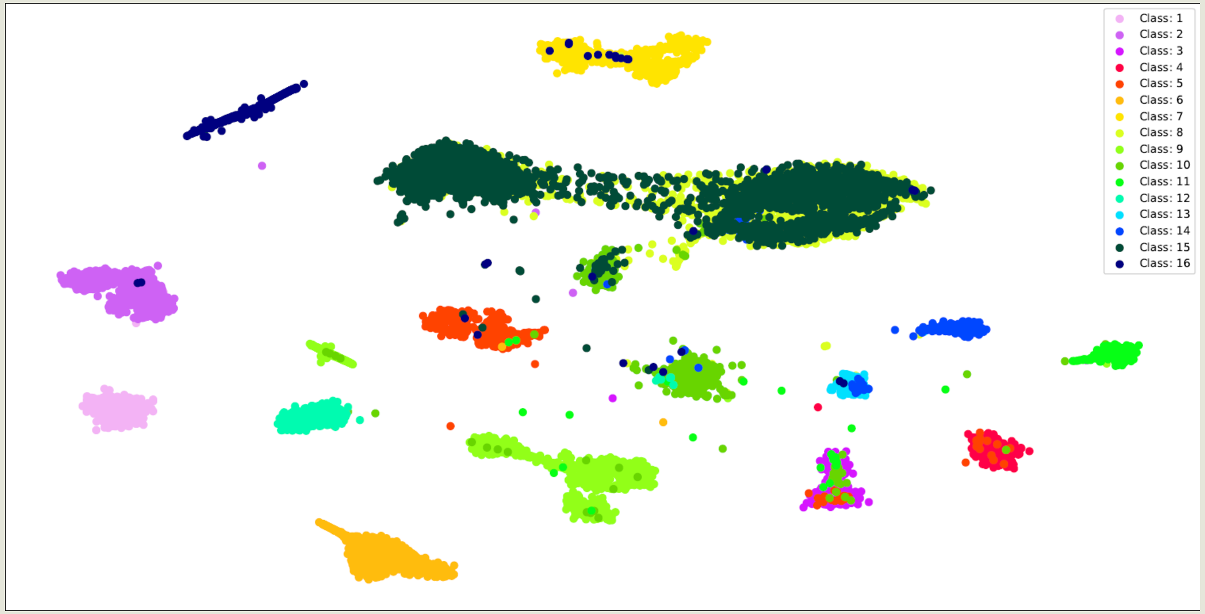
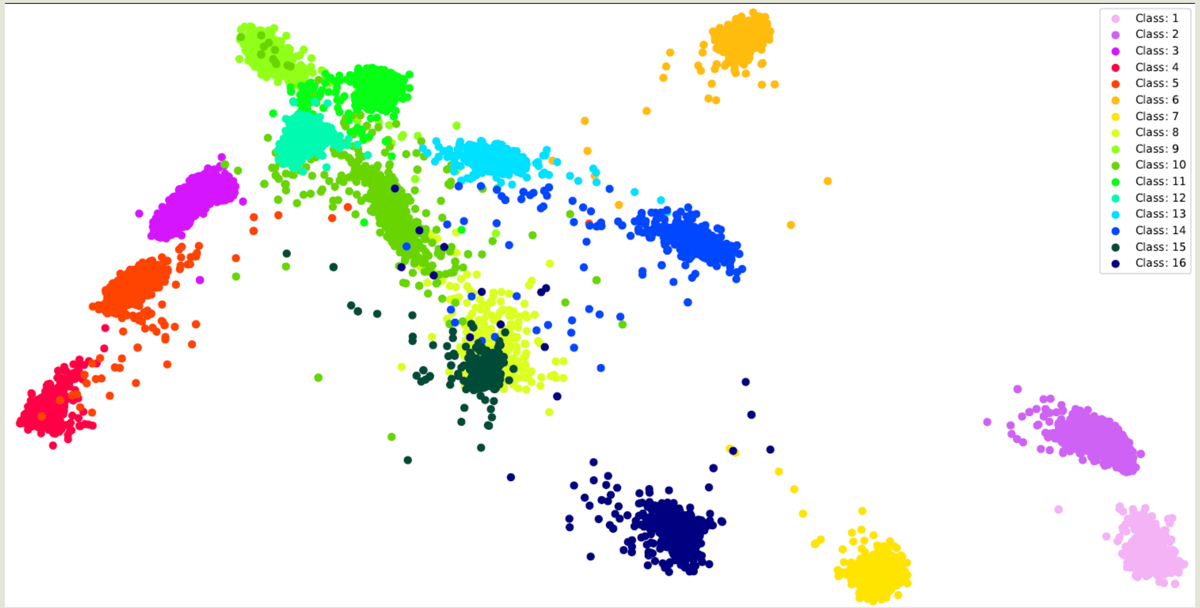
Experimentation

Clustering With Labels

Metric	**LEt-SNE** (sup)	UMAP (sup)	Autoencoder	PCA	UMAP (unsup)
SVM (OA)	**0.9286**	0.899	0.8358	0.8296	0.8524
Kappa (κ)	**0.9234**	0.8876	0.8178	0.811	0.8361





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Experimentation
Clustering With Labels

Confusion Matrix_SVM: UMAP

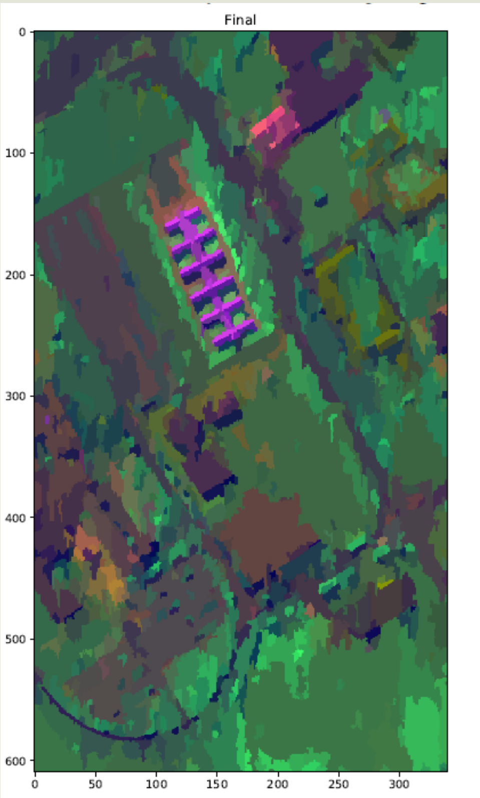
True label	Alfalfa	Corn-notill	Corn-mintill	Corn	Grass-pasture	Grass-trees	Grass-pasture-mowed	Hay-windrowed	Oats	Soybean-notill	Soybean-mintill	Soybean-clean	Wheat	Woods	Buildings-Grass-Trees-Drives	Stone-Steel-Towers
Alfalfa	0.08	0.00	0.00	0.00	0.00	0.00	0.46	0.46	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Corn-notill	0.00	0.64	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.10	0.20	0.03	0.00	0.00	0.00	0.00
Corn-mintill	0.00	0.17	0.48	0.04	0.01	0.00	0.00	0.00	0.00	0.01	0.25	0.03	0.00	0.00	0.00	0.00
Corn	0.00	0.23	0.17	0.34	0.03	0.09	0.00	0.00	0.00	0.03	0.09	0.03	0.00	0.00	0.00	0.00
Grass-pasture	0.01	0.00	0.00	0.00	0.88	0.02	0.00	0.01	0.00	0.02	0.00	0.01	0.01	0.02	0.00	0.00
Grass-trees	0.00	0.00	0.00	0.00	0.01	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.01	0.00
Grass-pasture-mowed	0.00	0.33	0.00	0.00	0.00	0.00	0.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hay-windrowed	0.00	0.00	0.00	0.00	0.01	0.00	0.03	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Oats	0.00	0.33	0.00	0.00	0.00	0.33	0.00	0.00	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Soybean-notill	0.00	0.03	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.83	0.08	0.02	0.00	0.00	0.00	0.00
Soybean-mintill	0.00	0.10	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.07	0.78	0.00	0.00	0.00	0.00	0.00
Soybean-clean	0.00	0.26	0.04	0.01	0.04	0.00	0.00	0.00	0.00	0.10	0.08	0.47	0.00	0.00	0.00	0.00
Wheat	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.98	0.00	0.00	0.00
Woods	0.00	0.00	0.00	0.00	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.96	0.02	0.00
Buildings-Grass-Trees-Drives	0.00	0.01	0.00	0.00	0.10	0.22	0.00	0.00	0.00	0.00	0.03	0.00	0.17	0.31	0.17	0.00
Stone-Steel-Towers	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

Confusion Matrix_SVM: LEt_SNE

True label	Alfalfa	Corn-notill	Corn-mintill	Corn	Grass-pasture	Grass-trees	Grass-pasture-mowed	Hay-windrowed	Oats	Soybean-notill	Soybean-mintill	Soybean-clean	Wheat	Woods	Buildings-Grass-Trees-Drives	Stone-Steel-Towers
Alfalfa	0.54	0.00	0.00	0.15	0.00	0.00	0.08	0.23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Corn-notill	0.00	0.54	0.16	0.08	0.00	0.00	0.00	0.00	0.00	0.01	0.07	0.13	0.00	0.00	0.00	0.00
Corn-mintill	0.00	0.15	0.57	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.17	0.00	0.00	0.00	0.00
Corn	0.00	0.26	0.20	0.40	0.00	0.00	0.00	0.00	0.03	0.00	0.03	0.06	0.00	0.00	0.03	0.00
Grass-pasture	0.00	0.00	0.01	0.02	0.86	0.02	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.06	0.00
Grass-trees	0.00	0.00	0.01	0.02	0.03	0.83	0.00	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.05	0.00
Grass-pasture-mowed	0.33	0.00	0.00	0.00	0.00	0.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Hay-windrowed	0.12	0.00	0.00	0.00	0.00	0.00	0.06	0.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Oats	0.00	0.00	0.00	0.33	0.00	0.33	0.00	0.00	0.33	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Soybean-notill	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.77	0.19	0.01	0.00	0.00	0.00	0.00
Soybean-mintill	0.00	0.01	0.11	0.01	0.00	0.00	0.00	0.00	0.00	0.09	0.74	0.03	0.00	0.00	0.00	0.00
Soybean-clean	0.00	0.13	0.16	0.05	0.00	0.01	0.00	0.00	0.00	0.01	0.07	0.57	0.00	0.00	0.00	0.00
Wheat	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.96	0.00	0.02	0.00
Woods	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.88	0.11	0.00
Buildings-Grass-Trees-Drives	0.00	0.00	0.01	0.00	0.04	0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.12	0.81	0.00
Stone-Steel-Towers	0.00	0.00	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.00	0.00	0.62

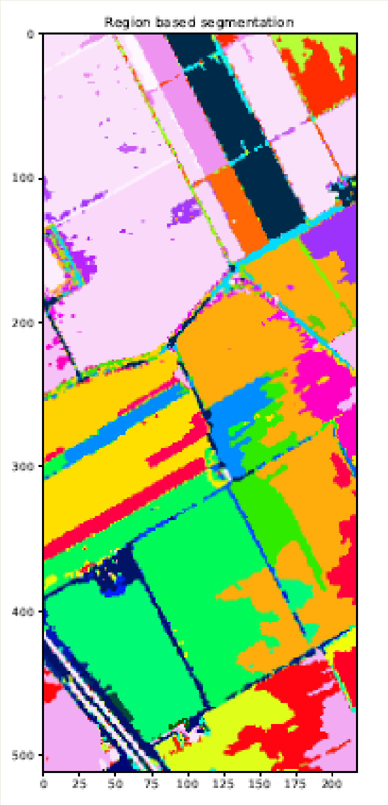
Experimentation
Clustering Without Labels

SLIC + RAG segmentation

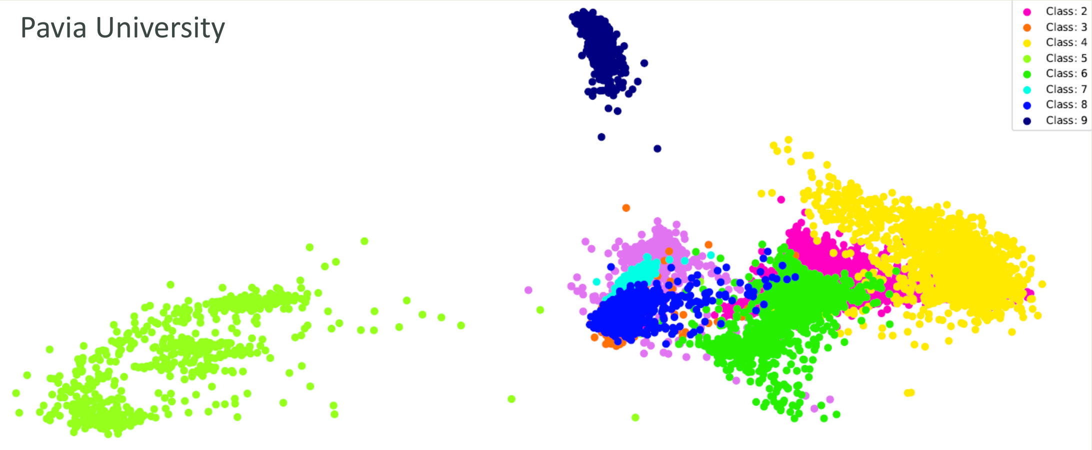


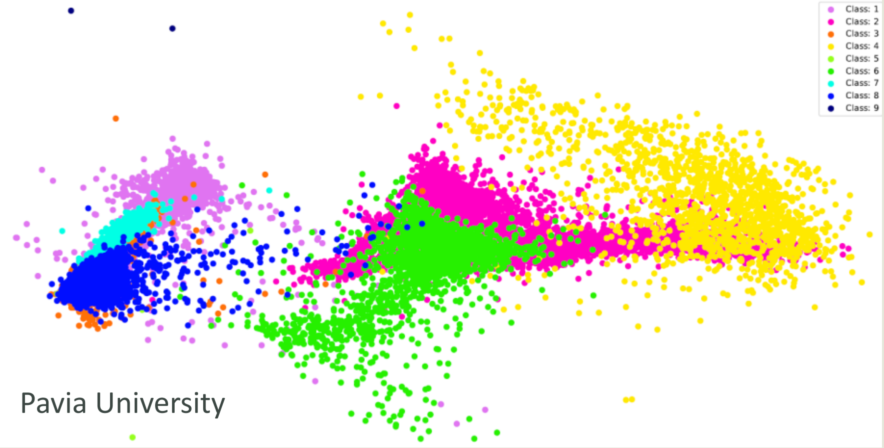
Pavia University

Watershed segmentation



Salinas





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Experimentation

Clustering

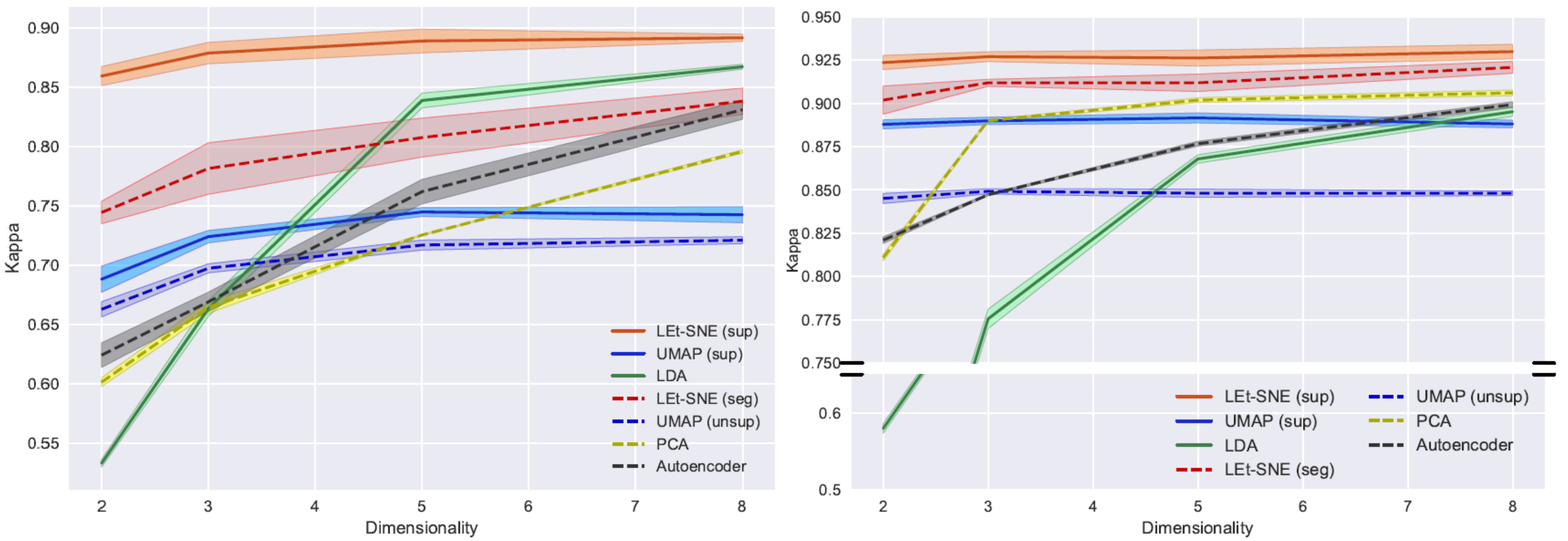

Fig. 3: Pavia (left) and Salinas (right): Comparing various supervised and unsupervised approaches

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Thank You!

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